3D GEOMETRY UNIT B:
VOLUME and CAPACITY

Topic	Assignment #	Date
Prisms and cylinders (p. 3-5) **Formulas (p. 4)**	1 (p. 6)	
Pyramids, Cones and Spheres **(p. 9-12)** **Formulas (p. 9-10)**	2 (p. 13)	
Volume and Capacity (p. 17-18)	3 (p. 19)	
Formulas, Unit conversion (p. 20-21)		

Name: _____ _____

Lesson 1: Volume

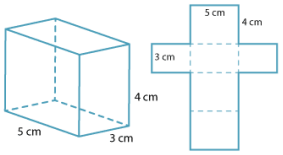
In this lesson, you will:

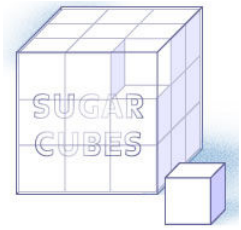
- Calculate volumes using both imperial and metric units
- Convert units of volume from imperial to imperial or metric to metric
- Manipulate a formula to calculate one dimension if the volume is given

Volume - Introduction

Volume and surface area are two different ways to describe 3-D objects.

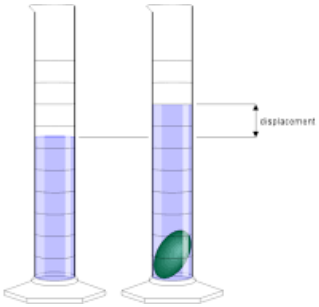
We saw in the previous booklet that **Surface Area** is the AREA of the **OUTSIDE** SURFACES of a 3-dimensional object, measured in **units²**.





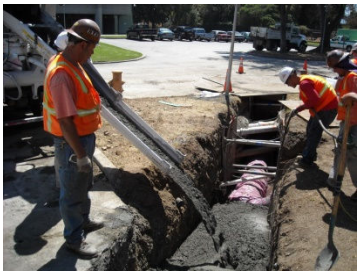
Volume is all of the SPACE **INSIDE** a three-dimensional object, or **the amount of space it occupies**, measured in **units³.**

-To find the volume, of a rectangular prism, you could count the number of cubes in it.



-Or if the shape is irregular and not easy to count cubes, you could place it in water. Measure how high the water is before the shape goes in and measure again after.

←The difference is the volume.



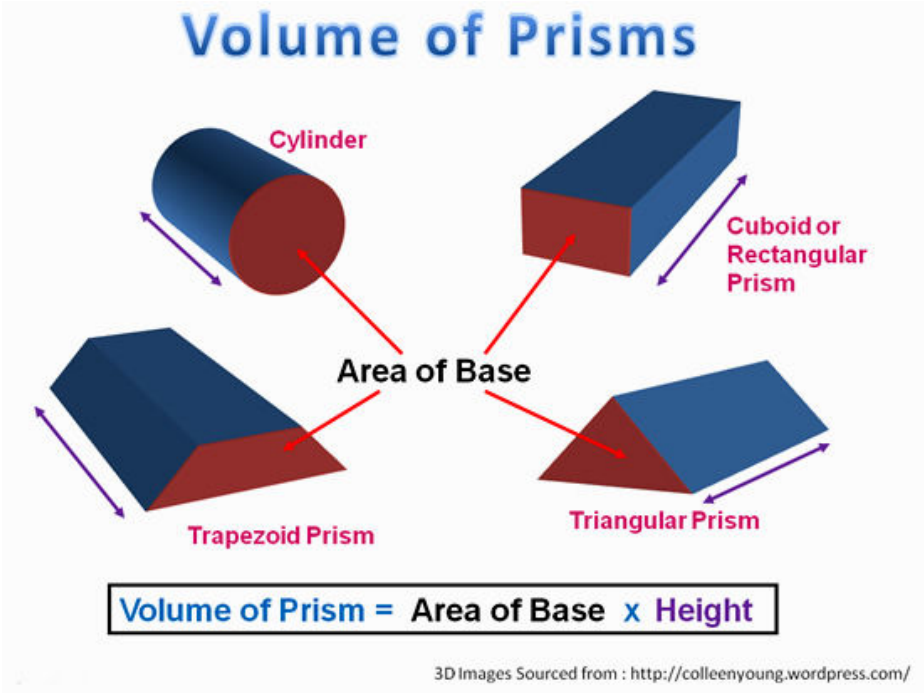
←Cement is delivered in m^3. To know how much cement should be ordered to fill a space, the workers must calculate the **volume** of the space.

Lesson 1: VOLUME (Prisms & Cylinders)

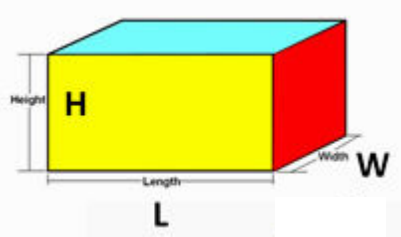
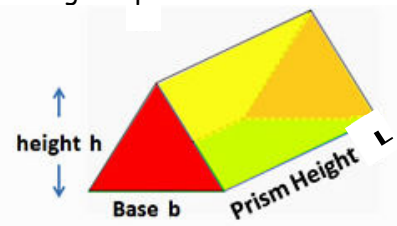
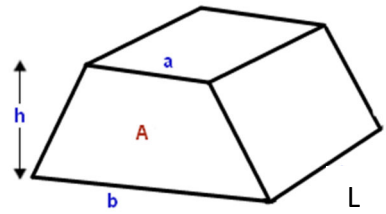
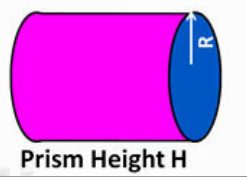
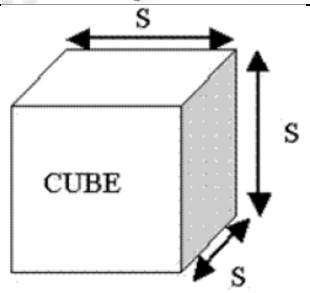
Volume of Prisms

But you can find the volume of any regular prism by **multiplying the area of the base by the height.**

(Remember a **prism** is a 3-D figure where the shape of the base is *consistent* throughout the object. The prism has 2 bases with the same shape. That shape is the **name of the prism**.)



Fill in the chart below – **Volume of Prisms** (*see last pages of this booklet for formula for area of 2-D shapes*)

prism	Name of base	Formula for area of 2-D base	Volume formula
Rectangular prism 			
Triangular prism 			
Trapezoidal prism  a,b -lengths of parallel lines of trapezoid (marked A) h – height of trapezoid L – length (height) of prism			
Cylinder 			
			

Examples: Volume of Prisms and Cylinders

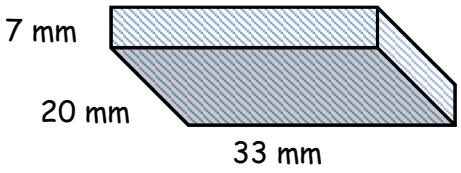
* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

1. Determine the total *volume* (in units^3) of the following **prisms**.

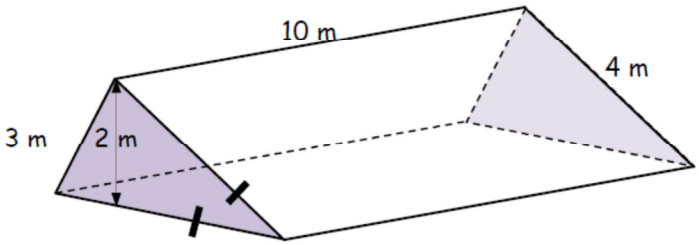
(If the answer is infinite decimal, round to 1 decimal place.) (formulas p. 4 and back of booklet)

The volume of a prism is: $V = \text{Area of Base} \times \text{Height}$

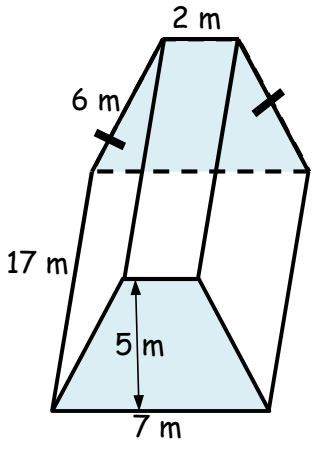
a) Rectangular Prism (4620 mm^3)



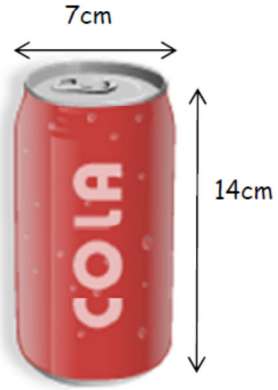
b) Triangular Prism (40m^3)



c) Trapezoidal Prism (382.5 m^3)



2. Determine how much cola could fit in the following container. (538.8 cm^3)
(Hint: find the *volume* of the following **cylinder**)



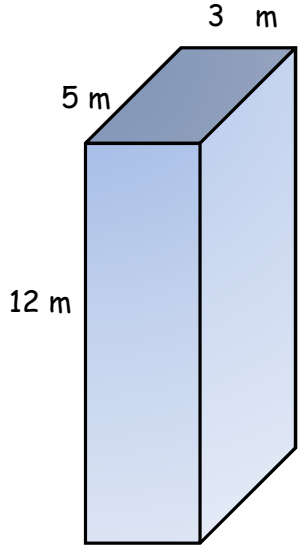
Assignment 1: VOLUME (Prisms & Cylinders)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

(solutions p. 8; formulas p. 4 and back of booklet)

1. Determine the total *volume* of the following **prisms**:

a)



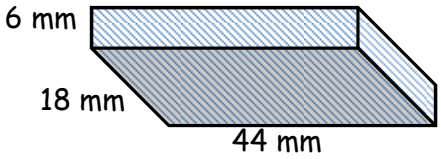
L = _____ W = _____ H = _____

V = _____

V = _____ × _____ × _____

V = _____

b)



L = _____ W = _____ H = _____

V = _____

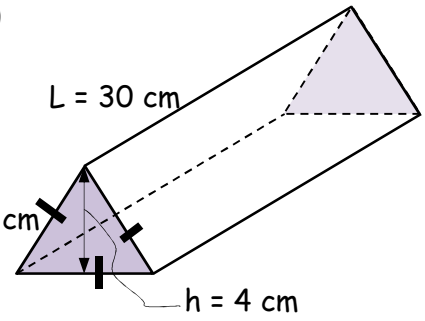
V = _____ × _____ × _____

V = _____

2. Determine the total volume of the following **prisms** (**triangular**):

b = _____ h = _____ L = _____

a)



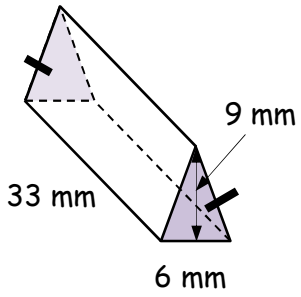
V = Area of base (triangle) x Height of prism

V = _____

V = _____

V = _____

b)



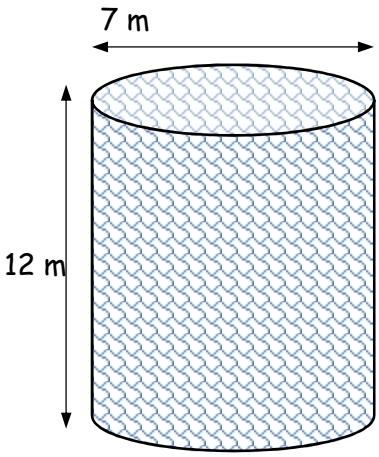
V = _____

b = _____ h = _____ H = _____

V = _____

V = _____

3. Determine the total *volume* of the following **prisms** (**cylinders**):



a) r = _____ h = _____

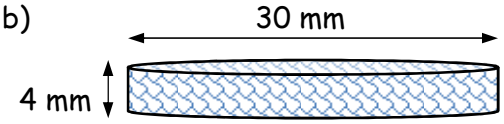
V = Area of base (circle) x Height of cylinder

V = _____

V = π x (_____)^2 x _____

V = _____

b)



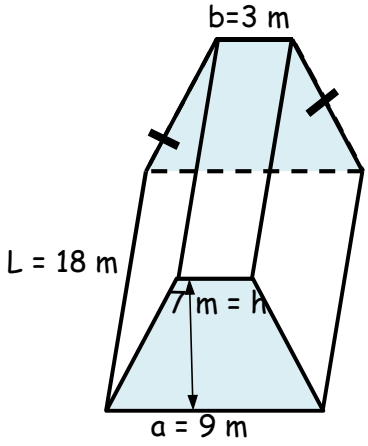
r = _____ h = _____

V = _____

V = π × (_____)^2 × _____

V = _____

4. Determine the total volume of these **trapezoidal** prisms.



V = Area of base (trapezoid) x Height of prism

V = _____

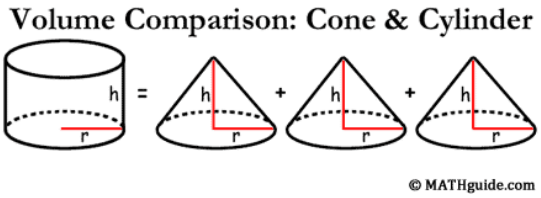
V = _____

V = _____

Answers: 1) 180 m^3, 4752 mm^3, 2) 300 cm^3, 891 mm^3
3) 461.8 m^3, 2827.4 mm^3 4) 756 m^3

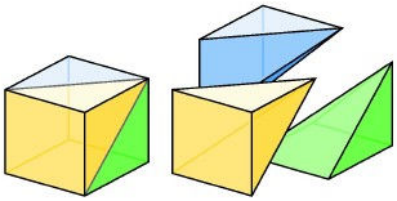
Lesson 2: VOLUME (Pyramids, Cones and Spheres)

Volume of Pyramids and Cones

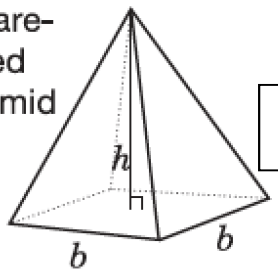


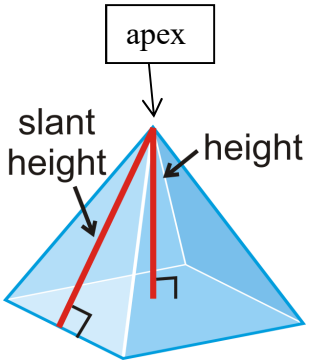
You can put 3 cones together to form a cylinder.

Similarly, you can put 3 square-based pyramids together to form a cube.



For that reason, the formulas for volume of a cone and a pyramid are 1/3 times the formula for cylinder and for cube.

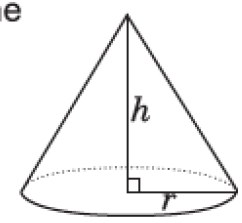
Square-based pyramid  Shape of base: square	$V = \frac{(\text{area of base})(\text{height})}{3}$ $V = \frac{1}{3}b^2h$ or $V = \frac{b^2h}{3}$

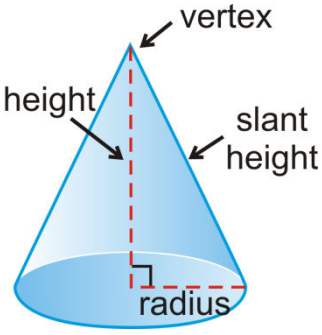


Note:

●For **surface area** of pyramid , we use the **slant height (s)** (*height of the triangle).*

●For **volume** of a pyramid, we use the **vertical height (h)** of the pyramid from the base to the apex *(if we were inside the pyramid, the height is from the floor straight up to the top point)*

Cone 	$V = \frac{(\text{area of base})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$



Similar to the height (h) and slant height (s) of pyramid,

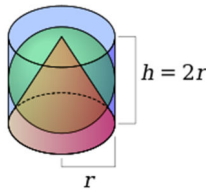
●the **height of the cone (h)** is the **vertical height** from the base of the cone to the vertex (top point) of the cone. The height is perpendicular to the radius (*the height and the radius form a 90° angle*).

●the **slant height (s)** is the distance from the circle base to the vertex

Volume of Sphere

(*In case you were wondering about the formula for the volume of a sphere…*

The volume of a cone and sphere fit together make a cylinder (assuming they are made to perfectly fit each other, so the height is twice the radius). The cone is 1/3 the volume of the cylinder and so the **sphere is 2/3 the volume of the cylinder**. If you make height = 2 x radius, you can come up with the formula for the volume of a sphere.



Volume = 2/3 * volume of cylinder

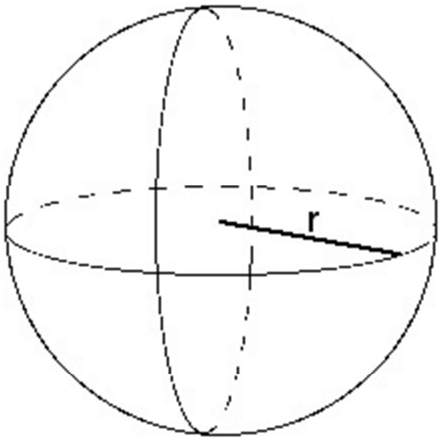
= 2/3 * $\pi(radius)^2 * height$

= 2/3 * $\pi(radius)^2 * (2 * radius)$

$= \frac{4}{3}\pi(radius)^3$

Therefore the formula for the

Volume of a Sphere is:



$$\text{Volume} = \frac{4}{3}\pi r^3$$

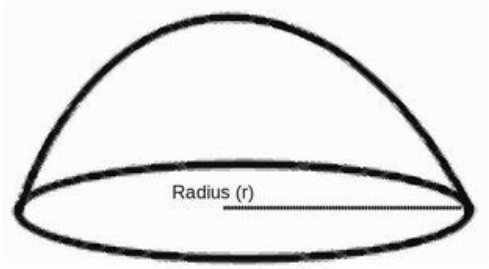
And then the volume of a hemisphere *(half a sphere)* is simply ½ times the volume of a sphere.

What is the VOLUME of a Hemisphere?

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2\pi}{3}R^3$$



Hemisphere Mathgun.com

Examples VOLUME - Pyramids, Cones and Spheres

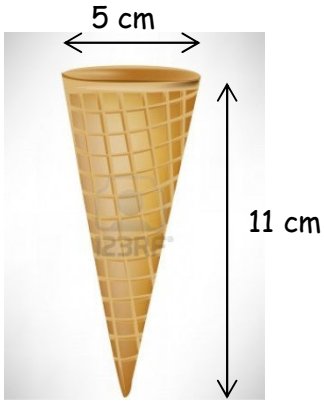
* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *
(formulas p. 9, 10 and back of booklet) (If the answer is infinite decimal, round to 1 decimal place.)

1. One of the famous pyramids built by the Egyptians is the great pyramid of Giza. Determine how much sand was used to build the pyramid (i.e. determine the total *volume* of the **pyramid**)
(2 574 466.7 m^3)

Pyramid Height = 146m

Square base length = 230m

2. Determine how much ice cream could fit into this cone before you put on the top scoop(s)
(i.e. determine the total volume of the **cone**) (28.8 cm^3)



3. Assume this water balloon is a perfect sphere. Determine the volume of water that is inside the balloon if its radius is 7cm. (1 436.8 cm^3)

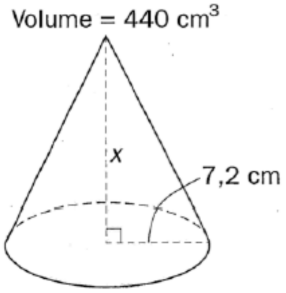
use exponent button on calculator
ie. y^x or x^y or ^

4. Find the volume of ice in this "snow cone" if the cone itself is also full.

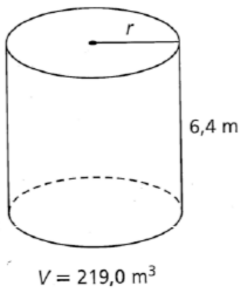
(see p. 10 – volume of hemisphere) (1 675 516.1 mm³)

The radius is 80 mm and the height of the cone is 90 mm.

5. Write out the formula for the volume of a cone and then substitute in the given values for volume and radius. Use algebra to solve to find the height of the cone, **rounded to 1 decimal place.** **(8.1 cm)**



6. Use a similar method as #5 to find the radius of this cylinder, rounded to 1 decimal place. (3.3 m)



Note $\sqrt{r^2} = r$ so when you are at the step of $r^2 = 10.9$, do the square root of both sides. This will result in r = ____

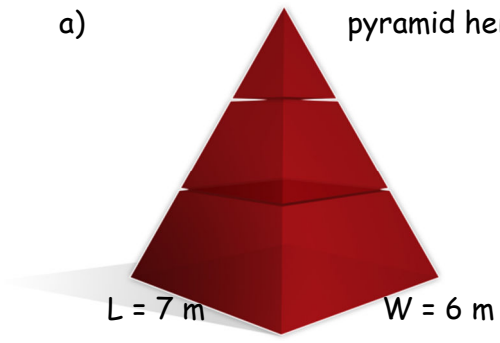
Assignment 2: VOLUME (Pyramids, Cones and Spheres)

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

(formulas p.4, 9, 10 and back of booklet) (If the answer is infinite decimal, round to 1 decimal place.) (solutions p. 15)

1. Determine the total *volume* of the following **pyramids**:

a) pyramid height (H) = 8 m



L = _____ W = _____ H = _____

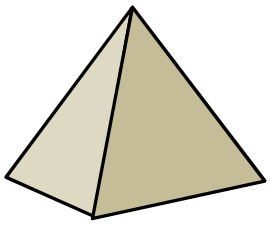
V = 1/3 Area of base (rectangle) x Height of pyramid

V = _____

V =_____

V = _____

b) Square base length = 7 in, height = 5 in



L = _____ W = _____ H = _____

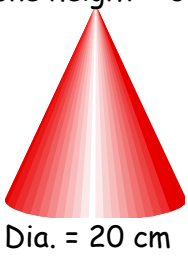
V = _____

V = _____

V = _____

2. Determine the total volume of the following **cones**:

a) cone height = 35 cm r = _____ H = _____



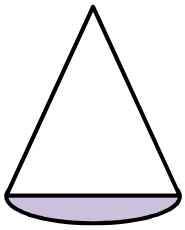
V = 1/3 Area of base (circle) x Height of cone

V = _____

V = _____

V = _____

b) Height = 70 yards, radius = 50 yd



r = _____ H = _____

V = _____

V = _____

V = _____

3. Determine the volume of the following **spheres**.

a) Radius = 1.5 dm

V = _____ r = _____

V = _____

V = _____

b) The diameter of the ping-pong ball is 3.5 cm.

V = _____ r = _____

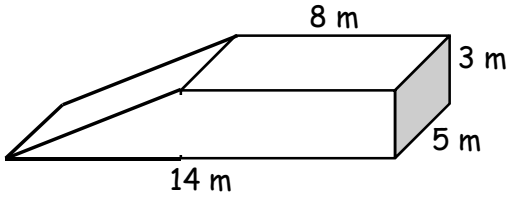
V = _____

V = _____

CHALLENGE!

4. Determine the volume of these *composite* objects (determine the volume of each part and add together).

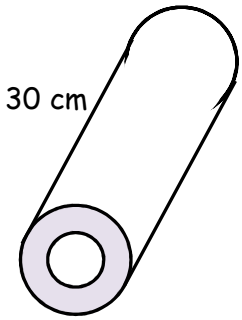
a)



Triangular prism	rectangular prism
B =	L =
H =	w =
L =	h =

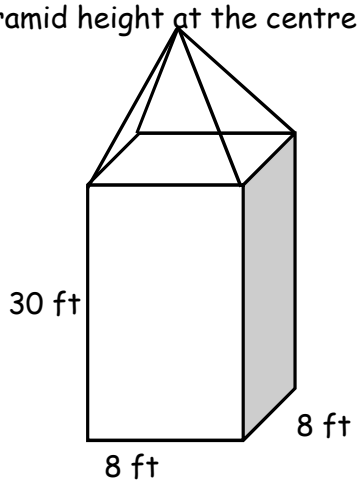
b) D_{out} = 9 cm, D_{in} = 5 cm (This is hollow. You'd need to subtract the area of the inner cylinder.)

R = r =



c) Pyramid height at the centre = 7 ft

Pyramid L= W= H= Prism L = W= H=



Answers: 1) 112 m^3, 81.7 in^3 2) 3665.2 cm^3, 183 259.6 yd^3
3) 14.1 dm^3, 22.4 cm^3 4) 165 m^3, 1319.5 cm^3, 2069.3 ft^3

See p. 12 for help..

5. The volume of a sphere is 41 m³. What is the <u>diameter</u> of the sphere? (4.4m)

1. On calculator find $4 \times \pi \div 3$.
2. Divide both sides by coefficient (number multiplying r^3).
3. Find cube root of both sides.

$\sqrt[3]{r^3} = r$, so if you find the cube root of both sides you'll find the radius.

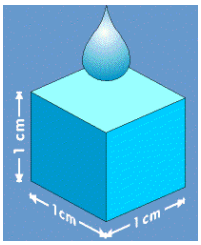
-To find a root on your calculator, use $\sqrt[x]{y}$ button on your phone. Press $\sqrt[x]{y}$ then 3 to find the cube root.

6. The volume of a rectangular prism is 720 mm³. If the width is 6 mm and the height is 10 mm, what is the length? **(12mm)**

Volume and Capacity

We have seen that the **volume** measures the **amount of space** inside an object.

Capacity is the amount of *liquid* that an object can contain *(the amount a container can hold)*. Capacity is usually measured in **millilitres (ml)** or **litres (l)** **(Common imperial units are ounces, gallons, pints, and in cooking -cups, tbsp, tsp).** The amount of ml in a can of soda or litres of milk is the **capacity of liquid** in that container – how much liquid it can hold. In everyday life we refer to capacity when cooking (how many cups, ounces), when buying food (how many ml is in the can), when buying gas (in litres), etc.



*A cylindrical can of pop has a **volume** of **375 cm³**, but is labeled as **375 ml** because it contains **liquid**.*

1 ml of liquid fits exactly in a cube 1cm x 1cm x1cm **(1cm³)**

1 cm³ = 1 millilitre
1000 ml = 1 litre.
1 000 cm ³ = 1 litre
1m³ = 1 kilolitre

To find capacity,

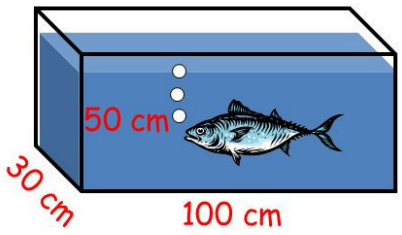
1. Find the volume in cm³.
2. Convert your cm³ answer to ml (same numbers; different unit).

- To find how many litres equal volume in cm³,
  - First convert cm³ to ml, (=) then convert ml to L (x1000).
    or:
  - Find the answer in m³, convert to kL (=), then convert kL to L (÷1000).

- To find out how many ml or litres in an object with dimensions in metres,
  - **1. C**onvert the dimensions to cm.
  - 2. Follow the steps above.

Example

How much water can this fish tank hold in litres?

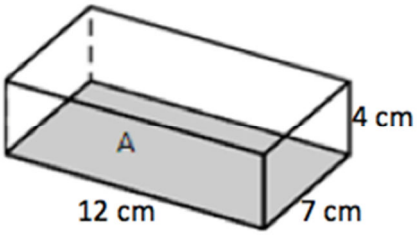
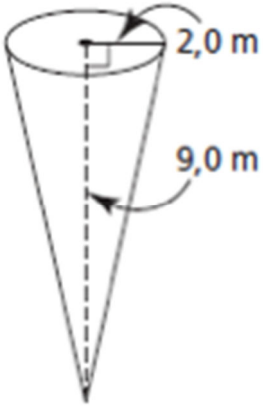


1cm³ = 1 mL
1000 ml = 1 litre
1 m³ = 1 kL

Method 1 (find cm³, convert to ml, convert to L)

Method 2 (convert dimensions to m, find m³, convert to kL, convert to L)

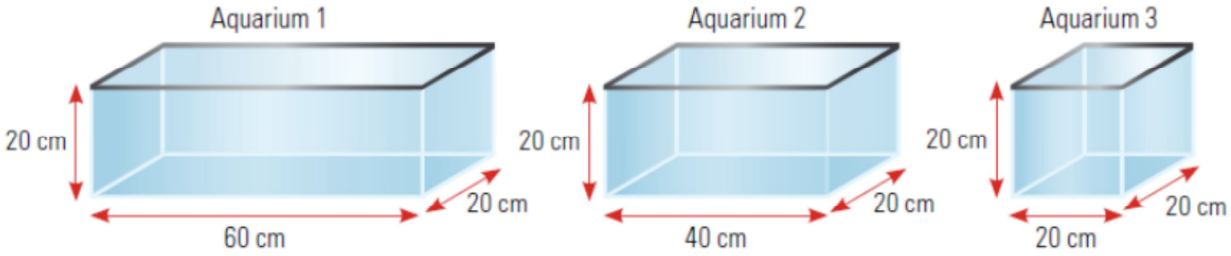
Try it!

1 A can of soup has a volume of 460 cm^3. What is the capacity of the can in ml? _____ What is the capacity in litres? _____	2 A plastic box measures 24 cm wide, 33 cm long, and 12 cm high. What is the capacity of the box, in litres?
3 What is the capacity of the reservoir, in ml?  	4 What is the capacity of the cone-shaped container, in litres? 

Assignment 3: VOLUME and CAPACITY

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

1. Aaliyah has 3 aquariums with the following dimensions. She needs to buy distilled water to fill the aquariums.



a) Calculate the total volume of the 3 aquariums, in cm^3.

b) What is the total capacity of the 3 aquariums together, in litres?

c) Aaliyah can buy 4 litre bottles of distilled water for $3.95 and 2 litre bottles of distilled water for $2.19, taxes included. Aaliyah wants to pay the least possible to fill the aquariums. How much would it cost to fill with the 4L or 2L bottles? What is the least she can pay to fill the aquariums with water?

Area of 2D objects: in units²

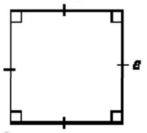
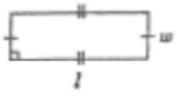
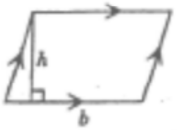
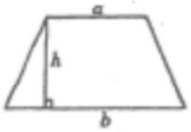
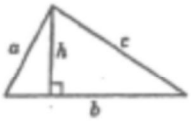
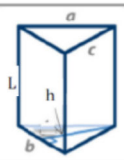
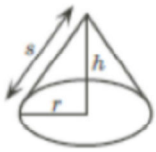
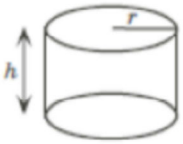
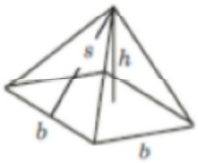
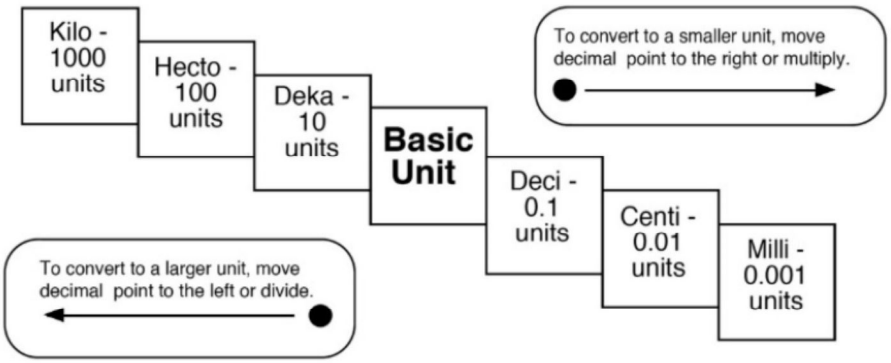
Name of Formula	Diagram	Formula
area of a square		$A = s^2$
area of a rectangle		$A = lw$
area of a parallelogram		$A = bh$
area of a trapezoid		$A = \frac{(a+b)h}{2}$ / $A = \frac{1}{2}(a+b)h$
area of a triangle		$A = \frac{bh}{2}$ / $A = \frac{1}{2}bh$
area of a circle		$A = \pi r^2$ (π button on calculator)

Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
rectangular prism		$SA = 2wh + 2lw + 2lh$ sides top/bottom front/back	$V = lwh$
sphere		$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
Triangular Prism		$SA = 2(\frac{bh}{2}) + aL + bL + cL$ bases sides (isoc. Δ → 2 sides same area)	$V = (\frac{bh}{2})L$
cone		$SA = \pi rs + \pi r^2$ (slanted side only)	$V = \frac{1}{3}\pi r^2 h$
cylinder		$SA = 2\pi rh + 2\pi r^2$ Side bases	$V = \pi r^2 h$
square base pyramid		$SA = b^2 + 2sb$ base sides (s - slant height h – vertical height from apex to base of pyramid	$V = \frac{1}{3}b^2 h$

1 foot = 12 inches

Metric Conversion Chart



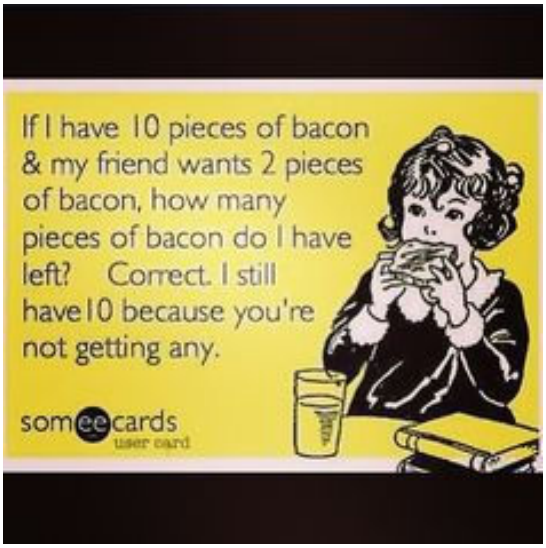
1 km = 1000 m
1 kg = 1000 g

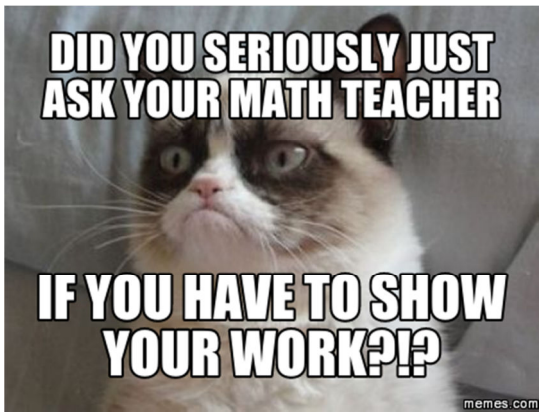
1 m = 100 cm
1 cm = 10 mm

1 g = 1000 mg
1 ℓ = 1000 ml

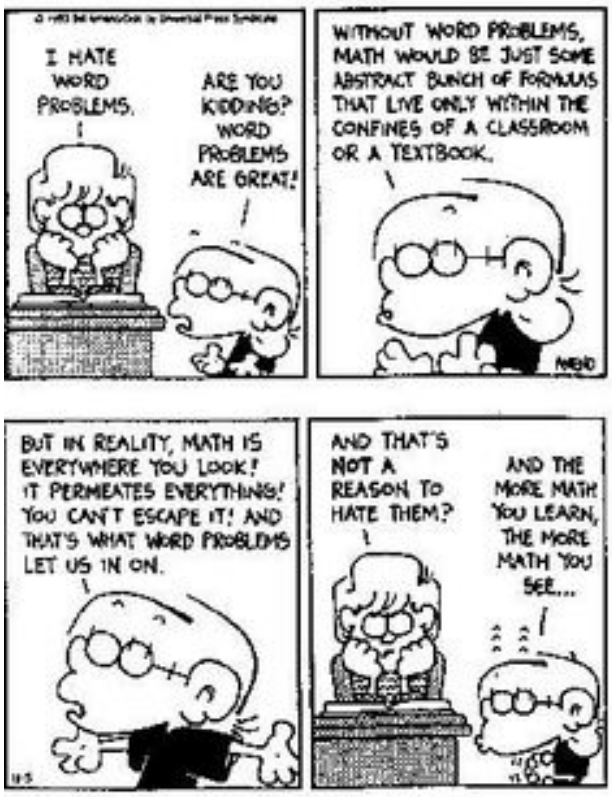
Math = Love

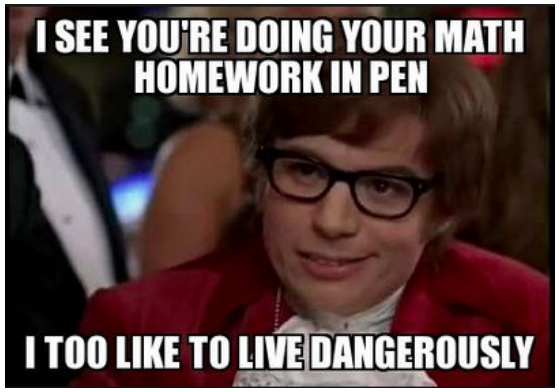
THINGS TEENAGERS SAY

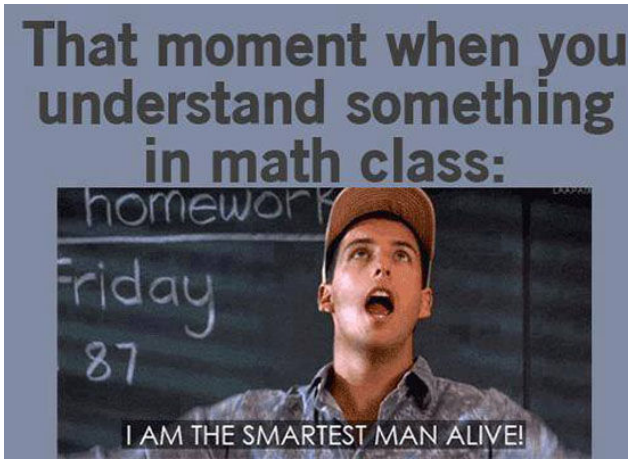




Fox Trot







MISTAKES *are proof that you are* TRYING